

ANALYSIS OF CONTINUOUS GIRDER BRIDGES WITH VARIABLE MOMENT OF INERTIA

BY

SABRI SAMI

B.Sc., Fouad I University, 1943
M.S., University of Illinois, 1951



AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING
IN THE GRADUATE COLLEGE OF THE
UNIVERSITY OF ILLINOIS, 1953

URBANA, ILLINOIS

1953

ANALYSIS OF CONTINUOUS GIRDER BRIDGES WITH VARIABLE MOMENT OF INERTIA

BY

SABRI SAMI

B.Sc., Fouad I University, 1943
M.S., University of Illinois, 1951

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING
IN THE GRADUATE COLLEGE OF THE
UNIVERSITY OF ILLINOIS, 1953

URBANA, ILLINOIS

1953

ACKNOWLEDGEMENT

To Thomas C. Shedd, Professor of Structural Engineering in the Department of Civil Engineering at the University of Illinois, under whose guidance this thesis was written, the writer is greatly indebted.

Professor Shedd's sympathetic understanding, friendship and kindly manner as well as his professional assistance are valued by the writer.

To Dr. Narbey Khachaturian, Assistant Professor of Civil Engineering, University of Illinois, whose suggestions were very helpful, the writer is grateful.

To Dr. Joseph Lee Waling, Associate Professor of Engineering Mechanics, Purdue University, Indiana, the writer gives thanks for advice in preparing the manuscript.

ANALYSIS OF CONTINUOUS GIRDER BRIDGES WITH VARIABLE MOMENT OF INERTIA

The analysis of continuous girder bridges with variable moment of inertia requires considerable work and often designers do not have sufficient time to make the detailed studies necessary to establish the most economical proportions. Several methods of expediting the analysis of continuous girder bridges have been developed in the past, but the task remained arduous and long.

In this thesis the writer presents what he considers to be an improved and accurate method of analysis for such structures. He believes it will materially reduce the labor required in methods previously published.

This work is confined to two-span and three-span continuous girder bridges with variable moment of inertia. The structures studied are of solid rectangular cross section whether a slab of width "b" or a girder of width "b". In this investigation the intrados follows a quadratic parabola, all the supports are hinged bearings and all but one are also assumed to be on horizontal rollers.

The principle of virtual work was used for the derivation of the elastic equations for the redundants for each structure studied. The magnitudes of these redundants are then expressed as the product of three factors as follows:

$$R = L \times C \times f(D)$$

in which

R = the redundant under consideration.

L = the "load factor". The "load factor" has the same dimension or units as the redundant under consideration and is a function of the external loads, dimensions and the material of the structure. It may be calculated from expressions given.

C = the "constant of the structure". The "constant of the structure" is a dimensionless quantity and is a function of the relative proportions of the structure only. It has a constant value for any particular structure and having been once determined, may be used in connection with analysis under any type or condition of loading.

$f(D)$ = the "D-function". This also is a dimensionless quantity but depends on the external loading as well as on the relative proportions of the structure.

In Chapter II a simple and rapid procedure for the analysis of the two-span continuous girder is considered. The structure is assumed to be symmetrical about the center support. In dealing with the two-span girder, the reaction at the center support was chosen as the redundant. It was found convenient to express the magnitude of the redundant as the product of two factors by writing the general formula

$$R = L \times C \times f(D)$$

as

$$R = L \times f(m).$$

Therefore, $f(m)$ is a dimensionless quantity but depends on the external loads and the relative proportions of the structure. Values of the function $f(m)$ are given in charts and tables for the following conditions of loading considered:

- (A) A uniformly distributed load " w " covering both spans.
- (B) A uniformly distributed load " w " covering either span.
- (C) A concentrated load " P " occupying any position on the span.
- (D) A continuous distributed load the intensity varying parabolically from zero at the end support to a maximum value at the center support.
- (E) A relative settlement, Δ , of one end support or of the center support.

The text of this chapter does not include the detailed mathematical derivations which led to the simple equations of the redundants suggested. Appendix A shows briefly the mathematical derivations involved, and the simplifying assumptions.

In Chapter III the analysis of the three-span continuous girder is considered. The structure is assumed to be symmetrical about the center of the middle span. In dealing with the three-span girder, the writer has found it to be most convenient to use the bending moments at the interior supports as the redundants. The general equations for these redundants are as follows:

$$M_B = L \times C_3 \times f(D)$$

$$M_C = L \times C_3 \times f(D)$$

in which

M_B = the bending moment at the support B

M_C = the bending moment at the support C.

The load factor L depends on the character of the load. When the load is a distributed load, L will be ωS^2 , or a fraction thereof. When the load is a concentrated load, the load factor will be PS . When settlement of supports is involved, the load factor will be some function of $\Delta EI/S^2$. In these expressions S is the span length.

The constant of the structure C_3 will appear in every expression for a redundant. As noted previously, it is a constant quantity for any particular structure. It is dimensionless and dependent solely on the relative proportions of the structure. Each three-span girder has its own particular constant.

The D-function, $f(D)$, is also dimensionless but is dependent on the character of the loading as well as on the relative proportions of the structure. The term "D-function" is a general term to describe a quantity made up of what may be called "D-factors" or "D-coefficients". The various D-func-

tions consist of one or more D-factors or coefficients and in most cases the ratio "n" of the length of the middle span to the length of the end spans also appears as a part of the D-function. The analyses require a total of eleven D-factors or coefficients designated by subscripts as D_1 to D_{11} . The values of these more or less complicated functions are given in charts and tables for the following conditions of loading considered:

- (A) A uniformly distributed load " w " covering the middle span.
- (B) A uniformly distributed load " w " covering either end span.
- (C) A concentrated load "P" occupying any position in an end span.
- (D) A concentrated load "P" occupying any position in the middle span.
- (E) A continuous distributed load the intensity of which varies parabolically.
- (F) A settlement, Δ , of an end support.
- (G) A settlement, Δ , of an interior support.

The text of this chapter does not include the detailed mathematical derivations which led to the simple equations of the redundants suggested. Appendix B shows briefly the mathematical derivations involved, and the simplifying assumptions.

It is believed that with the help of the data presented a designer will be able to make a sufficient number of studies of a bridge in a relatively short time to assure himself that the proportions finally chosen are economically as well as structurally adequate.

VITA

Sabri Sami was born in Cairo, Egypt, on July 15, 1919. After graduating from Fouad I Secondary School in 1938, he entered Fouad I University, Giza, Egypt. He was granted the degree of Bachelor of Science in 1943.

In August, 1943, he became an irrigation engineer in the Ministry of Public Works. In April, 1944, he was transferred to the Village Service Department, Ministry of Public Health, where he worked as a structural and construction engineer in the design and construction of water purification plants, water towers, hospitals and housing projects. At the same time, he worked for private companies in the design and construction of a water purification and softening plant of a capacity of 12.7 million gallons per day, the design of numerous structures for a large calcium nitrate plant to fit the requirements of the Chemical Construction Corporation of New York and other similar designs in reinforced concrete.

In 1949 he was selected by the Egyptian Government to study in the United States and in February, 1950, entered the Graduate College of the University of Illinois.

He received the degree of Master of Science in February, 1951. He held a University Fellowship in the academic year 1952-53.

He worked on the design of highway bridges for the Illinois Division of Highways in the University during the summer of 1952. He is currently engaged in research on reinforced concrete culverts in connection with a project being studied at the University of Illinois.

He is a member of the Egyptian Society of Civil Engineers and of Sigma Xi.

